$\boxed{3B}$ Continue

To summarize from Equations (2-4), (2-5), (3-3), (3-4) and (3-5) as Equations (3-6) through (3-15):

$$\frac{\partial Gain}{\partial TFG} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \frac{1.0}{TFG} \tag{3-6}$$

$$\frac{\partial Phase}{\partial TFG} = 0.0 \tag{3-7}$$

$$\frac{\partial Gain}{\partial b_1{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \mathrm{Re}\left[\frac{s}{N^1} \right] \tag{3-8}$$

$$\frac{\partial Phase}{\partial b_1{}^1} = \frac{W\ (180.0/\pi\ )}{N_p} \mathrm{Im}\left[\frac{s}{N^1} \right] \tag{3-9}$$

$$\frac{\partial Gain}{\partial b_0{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \mathrm{Re}\left[\frac{1.0}{N^1} \right] \tag{3-10}$$

$$\frac{\partial Phase}{\partial b_0{}^1} = \frac{W\ (180.0/\pi\ )}{N_p} \mathrm{Im}\left[\frac{1.0}{N^1} \right] \tag{3-11}$$

$$\frac{\partial Gain}{\partial a_1{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \mathrm{Re}\left[\frac{-s}{D^1} \right] \tag{3-12}$$

$$\frac{\partial Phase}{\partial a_1{}^1} = \frac{W\ (180.0/\pi\ )}{N_p} \mathrm{Im}\left[\frac{-s}{D^1} \right] \tag{3-13}$$

$$\frac{\partial Gain}{\partial a_0{}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g} \mathrm{Re}\left[\frac{-1.0}{D^1} \right] \tag{3-14}$$

$$\frac{\partial Phase}{\partial a_0{}^1} = \frac{W\ (180.0/\pi\ )}{N_p} \mathrm{Im}\left[\frac{-1.0}{D^1} \right] \tag{3-15}$$

Fig. 3B

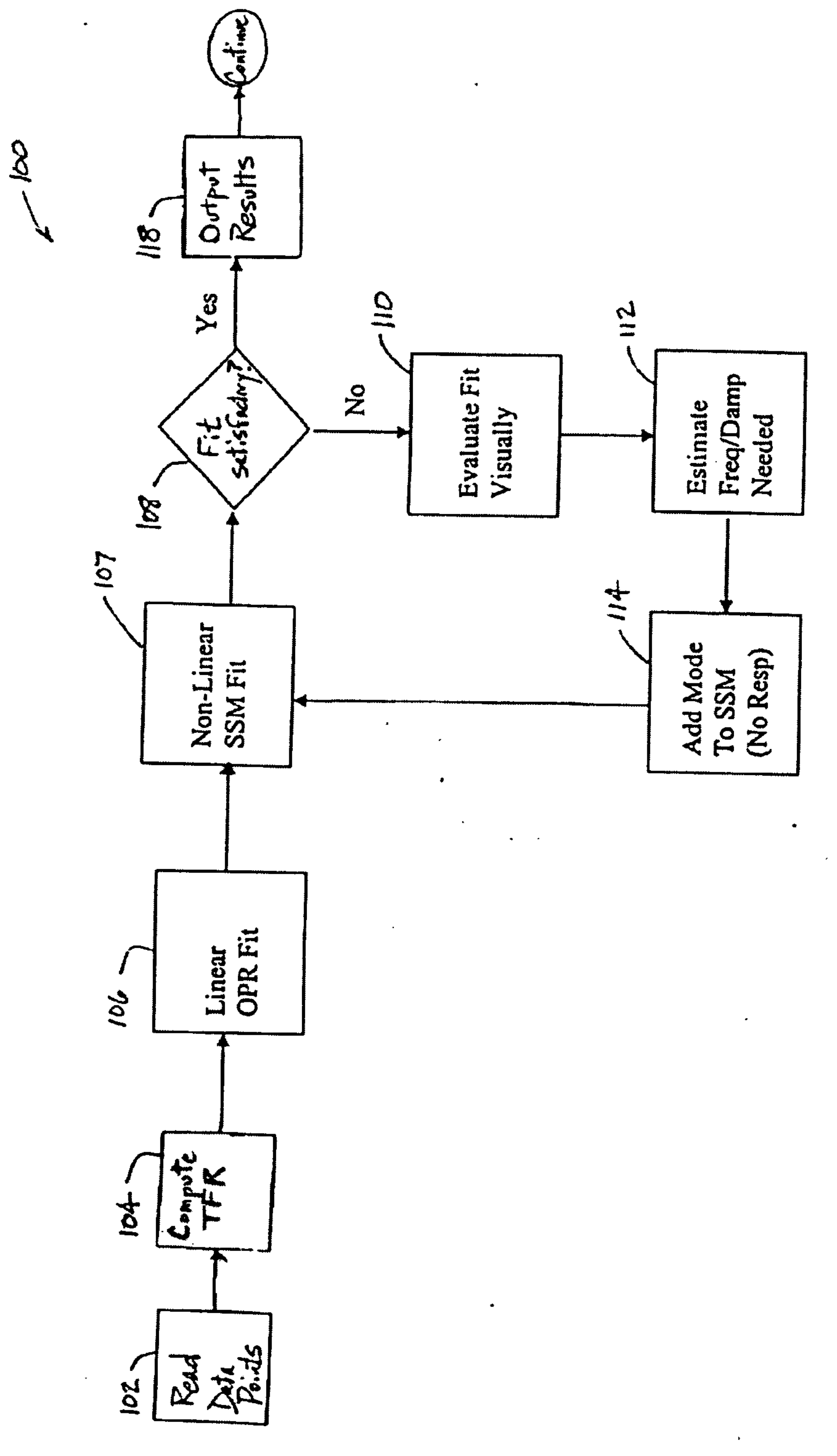
100
102
Read Data Points
104
Compute TFR
106
Linear OPR Fit
107
Non-Linear SSM Fit
108
Fit satisfactory?
Yes
No
110
Evaluate Fit Visually
112
Estimate Freq/Damp Needed
114
Add Mode To SSM (No Resp)
118
Output Results
Continue

Fig. 1

$$\frac{\partial Gain}{\partial x} = \frac{\partial\ (\ W/N_g\ 20.0\ \log_{10}(|Z|)\ )}{\partial x} \tag{2-1}$$

$$\frac{\partial Phase}{\partial x} = \frac{\partial\ (\ W/N_p\ (180.0/\pi)\ \tan^{-1}(Im(Z)/Re(Z))\ )}{\partial x} \tag{2-2}$$

Where: Gain = gain of transfer function response in dB
Phase = phase of transfer function response in degrees
W = frequency dependent weighting
Ng = gain normalization
Np = phase normalization
Z = complex transfer function frequency response
x = design variable

Since: $|Z| = \sqrt{Z\ Z^*}$

$\log_{10}(u) = \log_{10}(e)\ln(u)$

Gives: $20.0\log_{10}(|Z| = 10.0\log_{10}(e)\ln(Z\ Z^*)$

Then: $\frac{\partial Gain}{\partial x} = \frac{\partial\ (\ W/Ng\ 10.0\log_{10}(e)\ \ \ln(Z\ Z^*)}{\partial x}$

Since: $\frac{\partial \ln(u)}{\partial x} = \frac{1.0}{u}\frac{\partial u}{\partial x}$

$\frac{\partial \tan^{-1}(u)}{\partial x} = \frac{1.0}{1.0+u^2}\frac{\partial u}{\partial x}$

Then: $\frac{\partial Gain}{\partial x} = \frac{W\ 10.0\log_{10}(e)}{N_g\ \ (Re(Z)^2+Im(Z)^2)}\ \frac{\partial(Re(Z)^2+Im(Z)^2)}{\partial x}$

$\frac{\partial Phase}{\partial x} = \frac{W\ (180.0/\pi)\ Re(Z)^2}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\ \frac{\partial(Im(Z)/Re(Z))}{\partial x}$

Since: $\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left(v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}\right)$

Gives: $\frac{\partial Gain}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Re(Z)}{\partial x} + Im(Z)\frac{\partial Im(Z)}{\partial x}\right)$

$\frac{\partial Phase}{\partial x)} = \frac{W\ (180.0/\pi)}{N_p\ \ (Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Im(Z)}{\partial x} - Im(Z)\frac{\partial Re(Z)}{\partial x}\right)$

Continue to $\boxed{2B}$

Fig. 2A

2B Continue

There is similarity between the partial of the gain of the response and that of the phase. To uncover the similarity, examine Equation (2-3):

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{Re(Z)+Im(Z)j}\left[\frac{\partial Re(Z)}{\partial x} + \frac{\partial Im(Z)}{\partial x}j\right] \tag{2-3}$$

Gives:

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left[Re(Z)\frac{\partial Re(Z)}{\partial x} + Im(Z)\frac{\partial Im(Z)}{\partial x}\right] + \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left[Re(Z)\frac{\partial Im(Z)}{\partial x} - Im(Z)\frac{\partial Re(Z)}{\partial x}\right]j$$

Combining the results from Equations (2-1), (2-2) and (2-3) yield Equations (2-4) and (2-5):

$$\frac{\partial Gain}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_g}\,Re\left[\frac{1.0}{z}\frac{\partial z}{\partial x}\right] \tag{2-4}$$

$$\frac{\partial Phase}{\partial x} = \frac{W\ (180.0/\pi)}{N_p}\,Im\left[\frac{1.0}{z}\frac{\partial z}{\partial x}\right] \tag{2-5}$$

The complex response of the block diagonal SSM for a specific transfer function is given by Equation (2-6):

$$z_{ij} = \Sigma\left[\frac{N_{ij}^{\,l}}{D^l}\right] + d_{ij} \tag{2-6}$$

$$\text{Where: } N_{ij}^{\,l} = (c_{i1}^{\,l}\, b_{1j}^{\,l} + c_{i2}^{\,l}\, b_{2j}^{\,l})\, s + (c_{i2}^{\,l}\, b_{1j}^{\,l}\, a_{21}^{\,l} - c_{i1}^{\,l}\, b_{1j}^{\,l}\, a_{22}^{\,l} + c_{i1}^{\,l}\, b_{2j}^{\,l})$$

$$D^l = s^2 - a_{22}^{\,l}\, s - a_{21}^{\,l}$$

For elements in the D matrix the unknown term in Equations (2-4) and (2-5) is given by Equation (2-7) using Equation (2-6):

$$\frac{\partial z_{ij}}{\partial d_{ij}} = 1.0 \tag{2-7}$$

For elements in the A, B or C matrices, xl, the unknown term in Equations (2-4) and (2-5) is given by Equation (2-8):

$$\frac{\partial z_{ij}}{\partial x^l} = \frac{\partial\,(N_{ij}^{\,l}/D_l)}{\partial x^l} \tag{2-8}$$

Continue to 2C

Fig. 2B

$\boxed{2C}$ Continue

Since: $$\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left[v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x} \right]$$

Then: $$\frac{\partial z_{ij}}{\partial x^1} = \frac{1.0}{D^1 D^1}\left[D^1 \frac{\partial N_{ij}{}^1}{\partial x^1} - N_{ij}{}^1 \frac{\partial D^1}{\partial x^1} \right]$$

And thus: $$\frac{\partial D^1}{\partial c_{i1}{}^1} = \frac{\partial D^1}{\partial c_{i2}{}^1} = \frac{\partial D^1}{\partial b_{1j}{}^1} = \frac{\partial D^1}{\partial b_{2j}{}^1} = 0.0$$

Simplified: $$\frac{\partial z_{ij}}{\partial x_1} = \frac{1.0}{D^1}\left(\frac{\partial N_{ij}{}^1}{\partial x^1} \right) \quad \text{for } x^1 = c_{i1}{}^1, c_{i2}{}^1, b_{1j}{}^1, b_{2j}{}^1$$

From Equation (2-6) the non-zero partials of the block numerator and denominator are given as Equations (2-9) through (2-16):

$$\frac{\partial N_{ij}{}^1}{\partial c_{i1}{}^1} = b_{1j}{}^1 s + (b_{2j}{}^1 - b_{1j}{}^1 a_{22}{}^1) \tag{2-9}$$

$$\frac{\partial N_{ij}{}^1}{\partial c_{i2}{}^1} = b_{2j}{}^1 s + (b_{1j}{}^1 a_{21}{}^1) \tag{2-10}$$

$$\frac{\partial N_{ij}{}^1}{\partial b_{1j}{}^1} = c_{i1}{}^1 s + (c_{i2}{}^1 a_{21}{}^1 - c_{i1}{}^1 a_{22}{}^1) \tag{2-11}$$

$$\frac{\partial N_{ij}{}^1}{\partial b_{2j}{}^1} = c_{i2}{}^1 s + (c_{i1}{}^1) \tag{2-12}$$

$$\frac{\partial N_{ij}{}^1}{\partial a_{21}{}^1} = c_{i2}{}^1 b_{1j}{}^1 \tag{2-13}$$

$$\frac{\partial N_{ij}{}^1}{\partial a_{22}{}^1} = - c_{i1}{}^1 b_{1j}{}^1 \tag{2-14}$$

$$\frac{\partial D^1}{\partial a_{21}{}^1} = - 1.0 \tag{2-15}$$

$$\frac{\partial D^1}{\partial a_{22}{}^1} = - s \tag{2-16}$$

Continue to $\boxed{2D}$

Fig. 2C

$\boxed{2D}$ Continue

To summarize from Equations (2-4) through (2-16) as Equations (2-17) through (2-30):

$$\frac{\partial Gain_{ij}}{\partial a_{21}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{D^1 c_{i2}{}^1 b_{1j}{}^1 + N_{ij}{}^1}{D^1 D^1 Z_{ij}}\right] \quad (2\text{-}17)$$

$$\frac{\partial Phase_{ij}}{\partial a_{21}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{D^1 c_{i2}{}^1 b_{1j}{}^1 + N_{ij}{}^1}{D_1 D_1 Z_{ij}}\right] \quad (2\text{-}18)$$

$$\frac{\partial Gain_{ij}}{\partial a_{22}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{-D^1 c_{i1}{}^1 b_{1j}{}^1 + N_{ij}{}^1 s}{D^1 D^1 Z_{ij}}\right] \quad (2\text{-}19)$$

$$\frac{\partial Phase_{ij}}{\partial a_{22}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{-D^1 c_{i1}{}^1 b_{1j}{}^1 + N_{ij}{}^1 s}{D^1 D^1 Z_{ij}}\right] \quad (2\text{-}20)$$

$$\frac{\partial Gain_{ij}}{\partial b_{1j}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{c_{i1}{}^1 s + c_{i2}{}^1 a_{21}{}^1 - c_{i1}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}21)$$

$$\frac{\partial Phase_{ij}}{\partial b_{1j}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{c_{i1}{}^1 s + c_{i2}{}^1 a_{21}{}^1 - c_{i1}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}22)$$

$$\frac{\partial Gain_{ij}}{\partial b_{2j}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{c_{i2}{}^1 s + c_{i1}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}23)$$

$$\frac{\partial Phase_{ij}}{\partial b_{2j}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{c_{i2}{}^1 s + c_{i1}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}24)$$

$$\frac{\partial Gain_{ij}}{\partial c_{i1}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{b_{1j}{}^1 s + b_{2j}{}^1 - b_{1j}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}25)$$

$$\frac{\partial Phase_{ij}}{\partial c_{i1}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{b_{1j}{}^1 s + b_{2j}{}^1 - b_{1j}{}^1 a_{22}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}26)$$

$$\frac{\partial Gain_{ij}}{\partial c_{i2}{}^1} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{b_{2j}{}^1 s + b_{1j}{}^1 a_{21}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}27)$$

$$\frac{\partial Phase_{ij}}{\partial c_{i2}{}^1} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{b_{2j}{}^1 s + b_{1j}{}^1 a_{21}{}^1}{D^1 Z_{ij}}\right] \quad (2\text{-}28)$$

$$\frac{\partial Gain_{ij}}{\partial d_{ij}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ Re\left[\frac{1.0}{Z_{ij}}\right] \quad (2\text{-}29)$$

$$\frac{\partial Phase_{ij}}{\partial d_{ij}} = \frac{W\ (180.0/\pi\ )}{N_{pij}}\ Im\left[\frac{1.0}{Z_{ij}}\right] \quad (2\text{-}30)$$

Fig. 2D

The complex response of the PZM is given by Equation (3-1):

$$Z = \frac{N}{D} = \frac{TFG \prod N^1}{\prod D^1} \tag{3-1}$$

Where: $N^1 = s^2 + b_1{}^1 s + b_0{}^1$

$D^1 = s^2 + a_1{}^1 s + a_0{}^1$

The unknown term in Equations (2-4) and (2-5) is given by Equation (3-2) by using Equation (3-1):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{D\,N}\left[D\frac{\partial N}{\partial x} - N\frac{\partial D}{\partial x}\right] \tag{3-2}$$

The results of Equation (3-2) when the transfer function gain is the design variable, x, is given by Equation (3-3).

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{TFG} \quad \text{when } x = TFG \tag{3-3}$$

The results of Equation (3-2) when the a numerator block coefficient is the design variable, x, is given by Equation (3-4):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N}\left(\frac{\partial N}{\partial x}\right) = \frac{1.0}{N^1}\frac{\partial N^1}{\partial x} \quad \text{when } x = b_1{}^1 \text{ or } b_0{}^1 \tag{3-4}$$

Gives: $\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1}s$ when $x = b_1{}^1$

$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1}$ when $x = b_0{}^1$

The results of Equation (3-2) when the a denominator block coefficient is the design variable, x, is given by Equation (3-5):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D}\left(\frac{\partial D}{\partial x}\right) = -\frac{1.0}{D^1}\frac{\partial D^1}{\partial x} \quad \text{when } x = a_1{}^1 \text{ or } a_0{}^1 \tag{3-5}$$

Gives: $\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1}s$ when $x = a_1{}^1$

$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1}$ when $x = a_0{}^1$

Continue to 3B

Fig. 3A

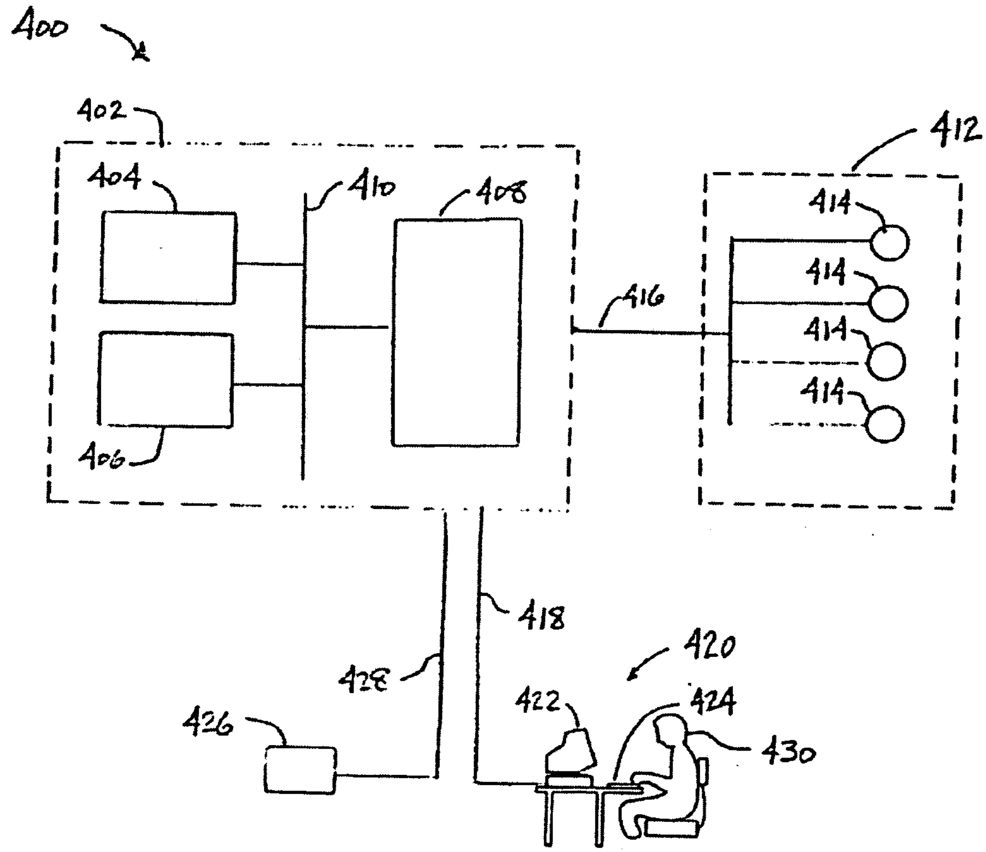
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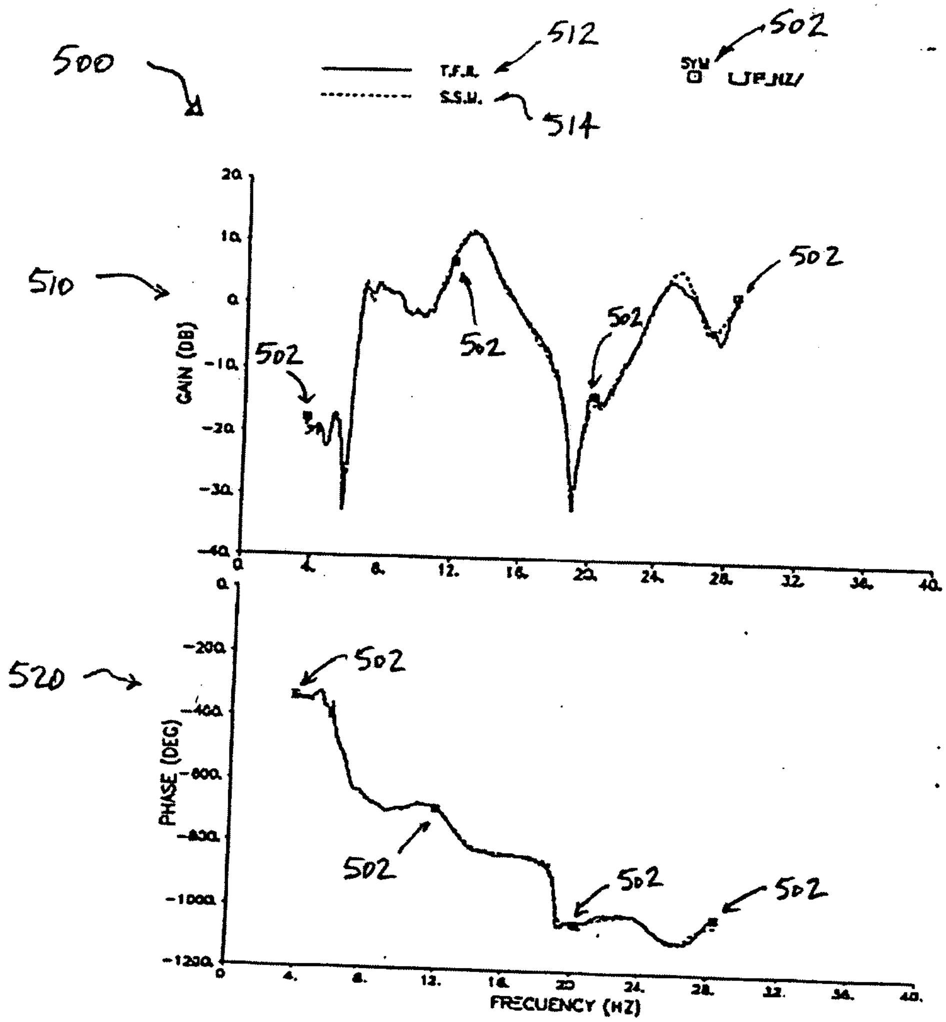
Fig. 4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T.F.R.
512
S.S.U.
514
SYM
502
510
GAIN (DB)
20.
10.
0.
-10.
-20.
-30.
-40.
0.
4.
8.
12.
16.
20.
24.
28.
32.
36.
40.
520
PHASE (DEG)
-200.
-400.
-600.
-800.
-1000.
-1200.
FRECUENCY (HZ)

Fig. 5